

Exercise 14

Prove the identity.

$$\tanh(x + y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$$

Solution

Use the definitions listed on page 259.

$$\begin{aligned} \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y} &= \frac{\left(\frac{\sinh x}{\cosh x}\right) + \left(\frac{\sinh y}{\cosh y}\right)}{1 + \left(\frac{\sinh x}{\cosh x}\right) \left(\frac{\sinh y}{\cosh y}\right)} \\ &= \frac{\frac{\sinh x}{\cosh x} + \frac{\sinh y}{\cosh y}}{1 + \frac{\sinh x \sinh y}{\cosh x \cosh y}} \times \frac{\cosh x \cosh y}{\cosh x \cosh y} \\ &= \frac{\sinh x \cosh y + \sinh y \cosh x}{\cosh x \cosh y + \sinh x \sinh y} \\ &= \frac{\sinh(x + y)}{\cosh(x + y)} \\ &= \tanh(x + y) \end{aligned}$$